AoPS Online
Art of Problem Solving

Syllabus — Scholars Math 9.1: Intermediate Counting & Probability

Scholars Math 9.1: Intermediate Counting & Probability discusses topics in discrete mathematics, including clever one-to-one correspondences, principle of inclusion-exclusion, generating functions, distributions, pigeonhole principle, induction, constructive counting and expectation, combinatorics, recursion, conditional probability, and introductory graph theory.

This course is specifically designed for high-performing students and draws material from many programs for top middle and high school students in the country. Our philosophy is that students develop more by learning to solve problems they haven't seen before, as opposed to offering repeated drills that students can memorize their way through. In this way, our classes are structured much more like courses at top-tier colleges.

Textbook(s): Scholars Math 9.1 requires *Intermediate Counting & Probability*, by David Patrick

Sample Problems:

- **[Principle of inclusion and exclusion]** 7 people are having a water balloon fight. At the same time, each of the 7 people throws a water balloon at one of the other 6 people, chosen at random. What is the probability that there are 2 people who throw balloons at each other?

- **[One-to-one correspondence]** A partition of a positive integer n is a decomposition of n into a sum of positive integers (not necessarily distinct), where we don't care about the order of the integers in the sum. Prove the number of partitions of n into exactly r parts is equal to the number of partitions of n in which the largest term is r.

- **[Generating functions]** I'm doling out 100 identical pieces of candy to 5 kids. The two youngest kids want at most 1 piece. The middle kid will take any number of pieces. The two oldest kids each demand an odd number of pieces. In how many ways can I distribute the candy?

- **[Graph theory]** Suppose in the freshman class at my university, if I choose any 4 students, at least one of the four knows all of the other three. (Assume that "knowing" is symmetric: if A knows B, then B knows A.) Prove that there must be a student that knows everybody in the class.

Common Core State Standards:

Domain	Subdomain	Standards
Algebra	Seeing Structure in Expressions	4
	Arithmetic with Polynomials & Rational Expressions	4,5
Functions	Building Functions	2
Statistics	Using Probability to Make Decisions	2, 5ab, 6, 7

Time Commitment: 18 lessons, 1.5 in-class hours + 4–5 hours of homework per lesson.

Grading: 64% Short-Answer Challenge Problems, 32% Writing Challenge Problems (proofs), and 4% Class Participation.

Content:

Lesson	Scholars Topic
1	Review of Counting and Probability Basics
2	Principle of Inclusion & Exclusion
3	Advanced Inclusion & Exclusion
4	Constructive Counting
5	One-to-one Correspondences
6	One-to-one Correspondences Continued and Pigeonhole
7	Constructive Expectation
8	Distributions
9	Mathematical Induction and Fibonacci Numbers
10	Recursion and Catalan Numbers
11	Conditional Probability
12	Combinatorial Identities
13	Events with States
14	Generating Functions, Week 1
15	Generating Functions, Week 2
16	Graph Theory, Week 1
17	Graph Theory, Week 2
18	Bonus Topics and Challenging Problems